2. ANTENNA PARAMETERS

Parameters of aerials can be divided into three groups.

Parameters, which characterize the aerial by the radiation field, belong to the first group. They are the parameters, which are determined from the distribution of electromagnetic waves in space. These parameters create the group of radiation characteristics, namely: the directional characteristic (DC) and the directional diagram (DD), the polarization pattern of the aerial, the directivity factor (DF), etc.

To the second group we shall attribute parameters, which characterize the aerial by intensity of electromagnetic waves, which feed the aerial. The most important of these parameters are the input impedance, the resistance of losses and the wave impedance of the aerial.

The third group of parameters characterizes the aerial as the converter of one type of electromagnetic waves to another. In this group there are the efficiency, the gain factor of the aerial, the resistance or the conductivity of radiation, the antenna logarithmic decrement, the frequency characteristic, the bandwidth, the effective length, the effective area, the effective temperature of the aerial, etc.

2.1. Directivity diagram (pattern)

The directional characteristic and DD of the aerial are determined by the distribution of the radiation field in space. As it follows from (1.19), the radiation field of the aerial is described by four multipliers, one of which shows the dependence of the field intensity on coordinate angles. So, this multiplier characterizes the distribution of the field in space. The dependence intensity of the electromagnetic field on coordinate angles is referred to as DC. Generally, if DC defines the spatial dependence of field components E or H, designation $f(\theta,\varphi)$ is used. If the maximum of this function is reduced to unity, DC is considered normalized and designated as $F(\theta,\varphi)$. In the case when DC defines the spatial dependence of the angular density of the power, it is designated as $f_p(\theta,\varphi)$ or $F_p(\theta,\varphi)$. The angular density of power $p(\theta,\varphi)$ is equal to the ratio limit

$$p(\theta,\varphi) = \lim_{\Delta\Omega \to 0} \frac{\Delta P}{\Delta\Omega},$$

where ΔP is the flux of the electromagnetic energy for a time unit through an element of a solid angle $\Delta\Omega$.

As the radiation of the aerial is often examined in a far zone, the angular density of the power is proportional to the square of the field intensity.

It is obvious, that

$$F(\theta,\varphi) = A_N f(\theta,\varphi); \qquad (2.1)$$

$$F_p(\theta,\varphi) = A_N^2 f_p(\theta,\varphi); \qquad (2.2)$$

; (2.2)

$$F_p(\theta,\varphi) = F^2(\theta,\varphi), \qquad (2.3)$$

where A_N is the normalization factor: $A_N = 1/f_{\max}(\theta,\varphi)$.

Graphic representation of DC is referred to as the directivity diagram. As it follows from the DC definition and from (1.19), the amplitude of the field intensity or the power density in any direction

$$E(\theta,\varphi) = E_{\max} F(\theta,\varphi); \qquad (2.4)$$

$$p(\theta,\varphi) = p_{\max} F^2(\theta,\varphi); \qquad (2.5)$$

$$\Pi(\theta,\varphi) = \Pi_{\max} F^2(\theta,\varphi). \qquad (2.6)$$

Equation (2.6) is the consequence of formula (2.5), as the module of Poynting's vector is connected to the angular density of power

$$p(\theta,\varphi) = r^2 \Pi(\theta,\varphi),$$

where r is the radius of sphere, on which the surface distribution of the field is examined (distance from aerial to a point of observation, which is located in a far zone).

Taking into account complexity of construction of spatial DD, the aerial is characterized by two-dimensional DDs, which are the cross-sections of the spatial DD at $\theta = \text{const}$ or at $\varphi = \text{const}$. Very frequently mutually perpendicular planes, passing through the direction of the maximal radiation are chosen.

For aerials, which radiate linearly polarized waves, the basic planes coincide with planes E and H of the electromagnetic field. In plane E there are vector E and Poynting's vector Π. Plane H

contains the vector of intensity of the magnetic field H and Poynting's vector.

Two-dimensional DDs are mainly represented in the polar or the Cartesian system of coordinates. In Fig. 2.1*(a)* lobed pattern of the directed aerial is plotted in the polar coordinate system, and in Fig. 2.1*(b)* - in the Cartesian coordinate system. If in the lobed pattern one of the directional lobes is the largest one, it is called the major lobe; others are called the side lobes.

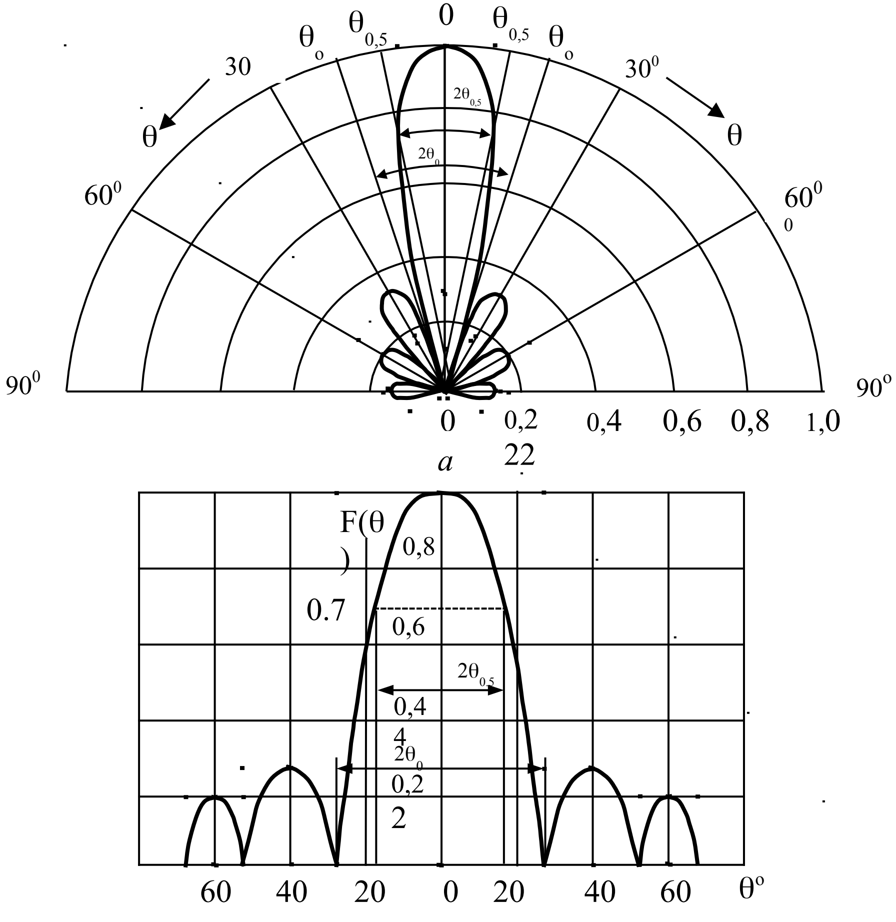

Fig.2.1

For convenience of comparison of aerials properties the beamwidth is used; it is defined by an angle between two directions with the identical intensity of radiation, which are symmetric with respect to the principal maximum (within the major lobe for the lobed pattern). Mainly, there are two levels of intensity, from which the beamwidth is determined. The first one is the level of the zero radiation. The width of the major lobe is determined from DD zeros and labelled $2\theta_0$. The second level is the level of density of the radiation power, equal to half of the maximal value. Thus beamwidth $2\theta_{0.5}$ is determined at level 0,5 from the maximal density of the field power [formulas (2.1) - (2.3)]. Beamwidth $2\theta_{0.5}$ is found between the directions, in which the intensity of the electric field is equal to 0,707 from the maximal value, as it follows from formulas (2.3) – (2.6).

The level of side lobes is defined as

$$\nu_p = \frac{E_{p\max}}{E_{\max}}, \tag{2.7}$$

where p is the number of a side lobe.

The level of side lobes may be determined in decibels

$$\nu_p = 20\lg\frac{E_{p\max}}{E_{\max}}. \tag{2.8}$$

2.2. Phase characteristic of directivity

Dependence of the field intensity phase on coordinate angles at constant distance from the aerial is referred to as the phase DC and designated $\psi(\theta,\varphi)$. Its graphic representation is the phase DD.

The phase DC enables to define the front of the electromagnetic wave, which is radiated by the aerial. It is known, that the wave front is a surface of equal phases. Therefore, equating in expression (1.18) the phase of the electromagnetic fluctuation $\psi(\theta,\varphi) - kr$ to any constant angle ψ_0, we find the equation of the wave front

$$r(\theta,\varphi) = \frac{\psi(\theta,\varphi) - \psi_0}{k}. \tag{2.9}$$

If function (2.9) describes the surface of the sphere, the aerial radiates spherical waves. The centre of such aerial coincides with the phase centre of the aerial.

2.3. Polarization characteristic

To define polarization characteristics of an electromagnetic wave the concept of the polarization plane is introduced. The polarization plane is defined by the direction of wave propagation and by the vector E orientation. For linearly polarized waves the plane of polarization does not change its position in space. While considering electromagnetic waves, propagated above the ground surface, one should divide the linearly polarized waves into horizontally polarized and vertically polarized waves. If the plane of polarization is perpendicular to the ground surface, the wave is called vertically polarized. Thus vector of the field intensity E is not necessarily perpendicular to the ground surface. If vector E is parallel to the ground surface, the wave is horizontally polarized. In this case the plane of polarization may be inclined under any angle to the ground surface.

If the polarization plane rotates around the propagation direction of the wave, the end of electric intensity vector, generally, describes an ellipse (Fig. 2.2). Such polarization is referred to as rotating or elliptic. The polarizing ellipse is characterized by either right or left direction of rotation. The right direction will be, when vector E turns counter-clockwise at wave propagation towards the observer, the left direction - at clockwise rotation of vector E. Polarization characteristics of the aerial are defined by an angle of inclination γ and the uniformity factor of polarization ellipse K_e. The inclination angle of the polarizing ellipse γ is the angle between the big axis of the ellipse and one of axes of the chosen coordinate system (Fig. 2.2).

The factor of uniformity or ellipticity of the polarization ellipse K_e is the ratio of small semi-axis of ellipse b to big semi-axis a :

$$K_e = \frac{b}{a}.$$

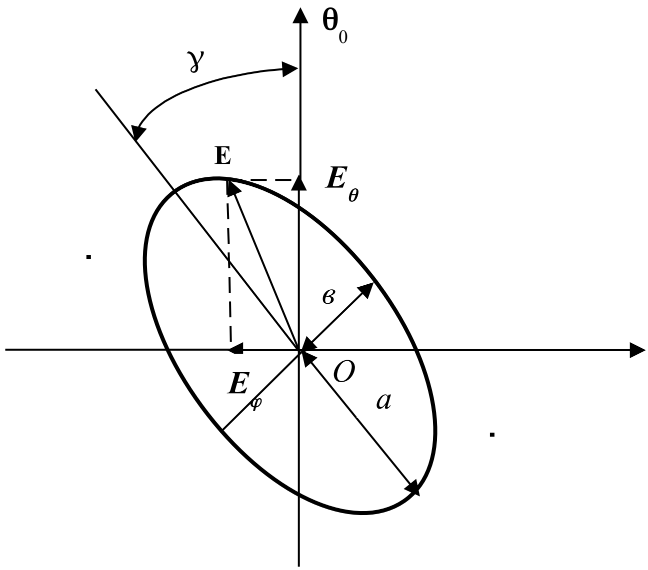

Fig. 2.2

For linear polarization $K_e = 0$, for circular - $K_e = 1$. If we assign the "+" sign to the right direction of rotation and "-" to the left one the factor of uniformity may take value in limits $-1 \le K_e \le 1$.

The elliptic polarization can be considered as a result of an interference of two linearly polarized waves (E_θ and E_φ in Fig. 2.2), polarization planes of which are mutually perpendicular and field intensity fluctuations are frequency identical, but phase shifted.

Directivity diagrams of elliptic polarized aerials are defined by the field distribution of linearly polarized waves of mutually perpendicular components E_θ and E_φ. Instant values of the electric field intensity of mutually perpendicular components may be written as

$$e_\theta = E_\theta \cos \omega t;$$
$$e_\varphi = E_\varphi \cos(\omega t + \psi), \qquad (2.10)$$

where E_θ, E_φ are amplitudes of intensity; ψ is the phase shift between vectors of intensity.

If to exclude function $\cos \omega t$ from expressions (2.10), the equation of the ellipse can be gained:

$$\frac{e_\theta^2}{E_\theta^2} - 2\cos\psi \frac{e_\theta e_\varphi}{E_\theta E_\varphi} + \frac{e_\varphi^2}{E_\varphi^2} = \sin^2\psi . \qquad (2.11)$$

Reducing equation (2.11) to the canonical form

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

turning coordinate system by angle γ from axis θ and replacing arguments in formulas

$$e_\theta = x\cos\gamma - y\sin\gamma \ \textit{and} \ e_\varphi = x\sin\gamma + y\cos\gamma,$$

we can find the connection between the parameters of the polarization ellipse and the parameters of mutually perpendicular components. As the factor at product xy is equal to zero, then

$$\operatorname{tg}2\gamma = \frac{2m\cos\psi}{m^2 - 1}.$$

The ellipticity factor may be found as b/a :

$$K_e = \pm\sqrt{\frac{m\sin^2\gamma - \sin 2\gamma\cos\psi + (1/m)\cos^2\gamma}{m\cos^2\gamma + \sin 2\gamma\cos\psi + (1/m)\sin^2\gamma}},$$

where $m = E_\theta / E_\varphi$ is the ratio of amplitudes of mutually perpendicular components.

According to the polarization characteristics of radiated waves the aerials are divided into the aerials of linear polarization and the aerials of rotating polarization. In the last class it is possible to distinguish aerials of elliptic polarization and aerials of circular polarization.

The polarization characteristic represents the dependence of the electromotive force (EMF) on terminals of the receiving aerial of linear polarization on the angle of turn of this aerial in a plane, perpendicular to a propagation direction of electromagnetic waves, which are radiated by the considered aerial. If to use Hertz dipole or dipole with the length, smaller than half of the wave length, in each position of the receiving aerial the EMF value will be proportional to the maximum value of the vector E projection on the longitudinal axis of the aerial.

In this connection the polarization characteristic at rotating polarization will slightly differ from the polarizing ellipse, being crossed with it in points, through which small and big axes pass. For linear polarization the polarization characteristic will look like figure of eight. And only in case of circular polarization the polarization characteristic will be a circle.

Except the polarization characteristic, in the aerial engineering concept of “polarization diagram” is used, which is the dependence of the ellipticity factor on coordinate angles $K_e(\theta,\varphi)$.

The factor of the polarization coordination is of big importance for the receiving aerial. It characterizes degree of conformity of polarization properties of the reception aerial with the polarization of an incidence wave. Quantitatively the factor of the polarization coordination is determined as the ratio of power $P_{P.D}$, which is absorbed in the matched loading of the receiving aerial at any polarization of the incidence flat wave, to power $P_{P.M}$, which is absorbed in the matched loading of the receiving aerial at incidence on it a flat wave with the same power density, under condition of the full polarization coordination:

$$a_{P.C} = \frac{P_{P.D}}{P_{P.M}} \qquad (2.12)$$

As it follows from expression (2.12), the factor of the polarization coordination is always positive, its minimal value may be equal to zero, the maximal value - to unity.

2.4. Resistance of radiation

From the theoretical electrical engineering it is known, that power that dissipates in an electric circuit with resistance R at flowing current I, is defined by the formula $P = 0.5 I^2 R$. It is obvious, that the radiation power can be calculated similarly:

$$P_\Sigma = \frac{1}{2} I^2 R_\Sigma , \qquad (2.13)$$

where R_Σ is the radiation resistance, that connects the square of the current in the aerial with the radiation power.

As the current in aerials is distributed non-uniformly, to eliminate uncertainty while choosing the current value we determine the radiation resistance from the current on the aerial terminal I_A

$$R_\Sigma = \frac{2P_\Sigma}{I_A^2}$$

(2.14)

or, at the current distribution in the aerial close to the sine law, from the current in antinode

$$R_{\Sigma m} = \frac{2P_\Sigma}{I_m^2}.$$

(2.15)

The radiation power of aerial P_Σ may be calculated by formula (1.27). In this case P_Σ is determined from the known value of Poynting's vector in a far zone.

On the aerial surface vectors E and H contain components with angle $\pi/2$ out of phase. Therefore, the calculated power will contain both real and imaginary components.

The real component is equal to the radiation power, which is defined by the method of Poynting's vector in a wave zone. The imaginary component is the power of the electromagnetic field near zone, which is connected to the aerial.

When the voltage distribution on the aerial is specified, the radiating ability of antenna is characterized by the conductivity of radiation, which represents the factor connected to the square of the voltage with the radiation power:

$$P_\Sigma = \frac{1}{2} G_\Sigma U_A^2.$$

(2.16)

Using (2.16) the conductivity of radiation as well as the resistance of radiation is defined with respect to the aerial terminal voltage

$$G_{\Sigma} = \frac{2P_{\Sigma}}{U_A^2}$$

or to the antinode voltage

$$G_{\Sigma m} = \frac{2P_{\Sigma}}{U_m^2}.$$

This parameter is applied to characterize slot antennas.

2.5. Efficiency

Efficiency of the aerial η_A is equal to the ratio of power P_{Σ}, radiated by the given aerial in the environment, to power P_A, feeding the aerial:

$$\eta_A = \frac{P_{\Sigma}}{P_A}.$$

The feeding power is spent on the radiation and compensation of losses, which occur because part of the feeding energy turns to heat in the aerial and is absorbed in the substances located in immediate proximity. Power to compensate losses is known as the power of losses. Losses in antenna devices are caused by following reasons:

- materials of the aerial have finite resistance, that is their conductivity differs from zero;
- any objects in the aerial near zone, will absorb a part of power (a mast, a ground surface, a covering of an aircraft, a tree, a building, etc.).

Thus, the feeding power P_A may be factored into two components: radiation power P_{Σ} and power of losses P_L. Taking this into account, we can set the expression for the radiation efficiency

$$\eta_A = \frac{P_{\Sigma}}{P_{\Sigma} + P_L}. \qquad (2.17)$$

Having divided the numerator and the denominator of the right part of expression (2.17) into the square of the current on the aerial terminals, we shall obtain

$$\eta_A = \frac{R_\Sigma}{R_\Sigma + R_L},$$

(2.18)

where R_L is the resistance of losses, which is defined through the power of losses and the current on the aerial terminals by the formula similar to expression (2.14).

2.6. Input impedance

The input impedance is the load for the aerial feeder. It may be calculated as the ratio of voltage on antenna terminals U_A to current I_A, which flows through terminals:

$$Z_A = \frac{\dot{U}_A}{\dot{I}_A} = R_A + \mathrm{i}\,X_A,$$

where R_A is the active component of the input impedance; X_A is the reactive component of the input impedance.

The input impedance of the aerial is very important, when we define the operation mode of the feeder and generator. The input power of the aerial is determined from the input impedance and the module of the input current of the aerial:

$$\widetilde{P}_A = \frac{1}{2} I_A^2 Z_A.$$

We can distinguish between active and reactive components of the input power:

$$\widetilde{P}_A = P_\Sigma + P_L + iP_r.$$

Having divided them into half of the square of the current module:

$$Z_A = R_\Sigma + R_L + iX_A.$$

Thus, the total input resistance consists of the radiation resistance and the resistance of the losses referred to the current on aerial terminals. The reactive component of the input impedance characterizes the reactive power of the field connected to the aerial in a near zone.

2.7. Directivity factor

There are some equivalent definitions of DF. So, DF is equal to the ratio of the angular power density in the given direction $p(\theta,\varphi)$, which is radiated by the considered aerial, to the angular power density in the same direction p_i, radiated by the reference aerial, under condition of equality of the radiation power:

$$D(\theta,\varphi) = \left[\frac{p(\theta,\varphi)}{p_{ref}}\right]_{P_\Sigma = P_{ref}} . \qquad (2.19)$$

The isotropic radiator, that emits electromagnetic waves of the identical intensity in all directions is supposed to be the reference aerial. Thus, expression (2.19) can be treated as the ratio of the angular power density, which is radiated in the given direction, to an average angular power density of radiation of the same aerial:

$$D(\theta,\varphi) = \frac{dP_\Sigma / d\Omega}{P_\Sigma / 4\pi}, \qquad (2.20)$$

where $dP_\Sigma / d\Omega = p(\theta,\varphi)$ is the angular power density at preset values θ and φ (in the given direction); $d\Omega$ is an element of a solid angle; $P_\Sigma / 4\pi = p_{ref}$ is the mean angular power density of radiation.

Rather frequently DF is the number, which shows how many times the radiation power of the reference aerial P_{ref} should be larger than the radiation power P_Σ of the examined antenna to provide the identical field intensity in a point of observation

$$D(\theta,\varphi) = \left(\frac{P_{ref}}{P_\Sigma}\right)_{E_{ref} = E(\theta,\varphi)} . \qquad (2.21)$$

As is follows from the above definitions, DF depends on the chosen direction. The real reference aerial is guided so that in the given direction its angular power density is maximal. Therefore in formulas (2.19), (2.20) the denominators do not depend on angular coordinates. The numerators of formulas (2.19) and (2.20) can be expressed with help of DC through power [formulas (2.4) - (2.6)] or as formula (2.5)

$$p(\theta,\varphi) = p_{\max} F_p(\theta,\varphi),$$

where $p_{\max}$ is the maximal angular power density of radiation.

Substituting formula (2.5) in expression (2.19), we obtain

$$D(\theta,\varphi) = \left[\frac{p_{\max}}{p_{ref}} F^2(\theta,\varphi) \right]_{P_\Sigma = P_{ref}}$$

or

$$D(\theta,\varphi) = DF^2(\theta,\varphi), \qquad (2.22)$$

where D is the DF in the direction of the maximal radiation.

On the basis of equations (2.19), (2.20) and (2.21) we can deduce expressions to calculate DF for any concrete aerial. We shall use formula (2.20) for this purpose. An element of a solid angle is defined as

$$d\Omega = \frac{dS}{r^2} = \sin\theta \, d\theta \, d\varphi,$$

where for the area element dS expression (1.25) is used.

The radiation power P_Σ is determined by formula (1.27). The derivative of this expression on the solid angle is equal to

$$p(\theta,\varphi) = \frac{dP_\Sigma}{d\Omega} = \frac{E_{\max} r^2 F^2(\theta,\varphi)}{2W}.$$

(2.23)

Let us substitute formulas for the angular power density (2.23) and the radiation power (1.27) in expression (2.20)

$$D(\theta,\varphi) = \frac{4\pi}{\int_0^{\pi}\int_0^{2\pi} F^2(\theta,\varphi) \sin\theta \, d\theta \, d\varphi} F^2(\theta,\varphi). \qquad (2.24)$$

DF, calculated in the direction of the maximal radiation, is the most frequently used. Thus, $F(\theta,\varphi) = 1$ and, consequently,

$$D = \frac{4\pi}{\int_0^{\pi}\int_0^{2\pi} F^2(\theta,\varphi) \sin\theta \, d\theta \, d\varphi}. \qquad (2.25)$$

In some cases the DF calculation is more convenient with the help of other formula. If the angular power density of the isotropic aerial (2.23) is known, a full radiation power is defined as the product of the angular power density (2.23) on a full spatial angle, which equals 4π steradian, that is:

$$P_{ref} = 4\pi\, p(\theta,\varphi) = \frac{4\pi E_{\max}^2 r^2 F^2(\theta,\varphi)}{2W}. \qquad (2.26)$$

Substituting values of the radiation power of the reference aerial (2.26) in formula (2.21)

$$D(\theta,\varphi) = \frac{2\pi E_{\max}^2 r^2 F^2(\theta,\varphi)}{W\, P_\Sigma}.$$

(2.27)

Taking into account, that for the free space $W = 120\pi\ \Omega$, expression (2.27) is resulted in

$$D(\theta,\varphi) = \frac{E_{\max}^2 r^2}{60 P_\Sigma} F^2(\theta,\varphi)$$

(2.28)

or for DF in the direction of the maximal radiation

$$D = \frac{E_{\max}^2 r^2}{60 P_\Sigma}.$$

(2.29)

2.8. Gain factor

The gain is equal to the ratio of the feed power of the reference aerial P_{ref} to the feed power of the considered aerial P_A under condition of equality of the radiation field intensity of both aerials in a point of observation:

$$G(\theta,\varphi) = \left(\frac{P_{ref}}{P_A}\right)_{E_{ref} = E(\theta,\varphi)}. \qquad (2.30)$$

For the reference aerial the efficiency is considered equal to unity. Taking into account, that $P_A = P_\Sigma / \eta_A$, and comparing expressions (2.21) and (2.30), at the same reference aerial

$$G(\theta,\varphi) = \eta_A D(\theta,\varphi). \qquad (2.31)$$

Taking into account obtained expression, formula (2.28) can be transformed to the expression for the gain determination:

$$G(\theta,\varphi) = \frac{E_{\max}^2 r^2}{60 P_A} F^2(\theta,\varphi). \qquad (2.32)$$

If antenna parameters and feeding power are known, then, by means of expression (2.32) it is possible to find the electric field intensity in a point of observation, which is located at distance r from the aerial in the direction, characterized by the meridional angle θ and the azimuthal angle φ:

$$E(\theta,\varphi) = \frac{\sqrt{60 P_A G(\theta,\varphi)}}{r}, \qquad (2.33)$$

where $E(\theta,\varphi) = E_{\max} F(\theta,\varphi)$.

Formula (2.33) is fundamental for field calculations and calculations of radiolines.

For the experimental determination of gain the dipole is often used as the reference aerial. In the range of ultrahigh frequencies a horn antenna is used as the reference aerial. Generally, the reference aerial in comparison with the isotropic radiator may be characterized by DF, different from zero

$$D_{ref} = \left(\frac{P_i}{P_{ref}} \right)_{E_i = E_{ref}}, \qquad (2.34)$$

where P_i is the radiation power of the isotropic radiator; P_{ref} is the radiation power of the reference aerial.

If DF of the aerial under study is calculated with respect to the isotropic radiator, then, as it follows from (2.21),

$$D(\theta,\varphi) = \left(\frac{P_i}{P_\Sigma} \right)_{E_i = E(\theta,\varphi)} = \left(\frac{P_{ref}}{P_\Sigma} \frac{P_i}{P_{ref}} \right)_{E_i = E(\theta,\varphi) = E_{ref}}.$$

Using (2.34), we obtain

$$D(\theta,\varphi) = D_{a.r}(\theta,\varphi) D_{ref},$$

where $D_{a.r}(\theta,\varphi)$ is the DF of the aerial, calculated with respect to the reference aerial with directivity D_{ref}.

Most frequently DF is determined by comparing to isotropic radiator. Therefore, DF with respect to an arbitrary chosen reference aerial

$$D_{a.r} = \frac{D(\theta,\varphi)}{D_{ref}}. \qquad (2.35)$$

Similarly to (2.35), for the gain factor

$$G_{a.r} = \eta_A \frac{D(\theta,\varphi)}{D_{ref}}.$$

For example, using the half-wave dipole as the reference aerial, we obtain

$$G_{a.r} = \eta_A \frac{D(\theta,\varphi)}{1.64},$$

as DF of the half-wave dipole with respect to the isotropic radiator is equal to 1.64.

2.9. Frequency properties of aerials

As the frequency of electromagnetic waves, feeding the aerial, varies, the distribution of currents, charges, tangential components of field intensity on the aerial surface, as well as the distribution of field intensity in space may also vary.

Thus, with the frequency change the aerial parameters will also alter. In many cases the reduction of the radiation power, changes of DC and DF can make the proper functioning of the radio channel impossible.

Generally, for an estimation of antenna frequency properties it is necessary to start with its amplitude and phase-frequency characteristics. The amplitude-frequency characteristic is the dependence of the relative amplitude of the radiation field intensity of the examined aerial in the observation point on the frequency of a feed current at its constant amplitude:

$$K(f) = \frac{E_f(\theta,\varphi)}{E_0(\theta,\varphi)}, \; r = \text{const},$$

where $E_f(\theta,\varphi)$ is the amplitude of the field intensity in a point with coordinates (r,θ,φ) at given value of frequency of the feed current on the aerial terminals; $E_0(\theta,\varphi)$ is the amplitude of the field intensity in the same point of observation at the certain average or resonant frequency of the feed current.

The phase-frequency characteristic is the dependence of the intensity phase of the aerial radiation field in the point of observation on the frequency of the feed current at its constant intensity:

$$\psi(f) = \arg \dot{E}_f(\theta,\varphi) - \arg \dot{E}_0(\theta,\varphi).$$

The distance from the aerial to observation point in this case is also supposed constant $(r = \text{const})$.

For maintenance of the reliable functioning of the radio engineering system in the given frequency range the values of antenna parameters should be in borders of specific tolerances. The opportunity of operation for the aerial in the given frequency range is determined by the dependence of its parameters on the frequency. Using such dependences and knowing allowable borders of parameters change, it is possible to establish the minimum $f_{\min}$ and the maximum $f_{\max}$ frequencies, which will limit a working frequency range of the aerial. The ratio $f_{\max}$ to $f_{\min}$ or of maximal working wavelength $\lambda_{\max}$ to minimal $\lambda_{\min}$ is referred to as the overlapping factor of the range:

$$K_r = \frac{f_{\max}}{f_{\min}} = \frac{\lambda_{\max}}{\lambda_{\min}}.$$

Another parameter, which characterizes frequency properties of the aerial, is bandwidth $\Delta f = f_{\max} - f_{\min}$. The bandwidth is a range of frequencies, in which limits changes of aerial parameters do not exceed the established tolerances. A relative bandwidth is determined by the formula

$$\delta f = \frac{\Delta f}{f_{av}} 100\% = 2\frac{f_{\max} - f_{\min}}{f_{\max} + f_{\min}} 100\%,$$

where f_{av} is the average frequency of a working range.

If $\delta f < 10\%$ the aerial is referred to as narrow-band or adjusted. Aerials, with the relative bandwidth greater than ten percent, are known as broad-band. Aerials, key parameters of which do not vary at frequency change in wider limits $(K_r > 4)$, are called frequency - independent.

2.10. Effective area

The parameter of the effective area mainly characterizes aperture aerials, which radiate electromagnetic waves through an aperture. The aperture plane of such aerials separates the internal volume of the aerial and the external space.

The ratio of the maximum power, which is generated in the matched load during the aerial operation in a receive mode, to the power density of the plane electromagnetic wave, which is incident on the aerial, is known as the effective area

$$S_e = \frac{P_{max}}{\Pi}, \qquad (2.36)$$

where P_{max} is the power, which is generated in the matched load; Π is the power density of a wave in an aperture plane.

As it will be shown further, the relation connects the effective area and DF

$$D = \frac{4\pi}{\lambda^2} S_e . \qquad (2.37)$$

As DF characterizes both transmitting and receiving aerials, the parameter "effective area" is used for the characteristic of receiving as well as transmitting aerials.

For the aperture aerials the ratio of the effective area to the geometrical area S is referred to as the area utilization factor:

$$\nu = \frac{S_e}{S}. \qquad (2.38)$$

The area utilization factor depends on the amplitude and phase distribution of tangential components of the electromagnetic field in the aperture plane.

2.11. Effective length

The field intensity of the radiator with the uniform current distribution in free space $(W = 120\pi\ \Omega)$ is

$$\dot{E} = i\frac{30kIl}{r}\sin\theta\, e^{-ikr}. \qquad (2.39)$$

In the direction of the maximum radiation $\theta = \pi/2$ the amplitude of the electric field intensity is equal to:

$$E_{\max} = \frac{30kIl}{r}. \quad (2.40)$$

Expression (2.40) is used to determine the effective length of the real aerial.

The effective length of the aerial is the length of the radia with the uniform current distribution, which in the direction of the maximum radiation creates the same value of the field intensity, as the examined aerial with the same current on terminals. If the value of current on aerial terminals I_A and the maximal value of the field intensity $E_{\max}$ are known, then, as it follows from expression (2.40), the effective length may be calculated as:

$$l_e = \frac{rE_{\max}}{30kI_A}. \quad (2.41)$$

By means of the effective length the radiation field of the real aerial can be found according to formula (2.39) for known DC and phase characteristic:

$$\dot{E}(\theta,\varphi) = i\frac{30kI_A l_e}{r} F(\theta,\varphi) e^{-ikr} e^{-i\psi(\theta,\varphi)}.$$

(2.42)

For the receiving aerial the other definition of the effective length is used. The effective length of the receiving aerial is equal to the ratio of EMF ε_A, which appears on terminals of the considered aerial, to the intensity of field E near the aerial:

$$l_e = \frac{\varepsilon_A}{E}. \quad (2.43)$$

In this case the aerial should be oriented so that EMF $\varepsilon_{\hat{A}}$ is maximal.

The values of the effective length, calculated by formulas (2.41) and (2.43) for the same aerial, coincide.

Let us substitute the maximal value of the field intensity from expression (2.41) in formula (2.29). Taking into account formula (2.14)

$$D = \frac{30k^2 l_e^2}{R_\Sigma}. \quad (2.44)$$

Expression (2.44) can be transformed for the calculation of the effective length

$$l_e = \sqrt{\frac{DR_\Sigma}{30k^2}}$$

(2.45)

or the radiation resistance on the known DF and effective length

$$R_\Sigma = \frac{30k^2 l_e^2}{D}. \qquad (2.46)$$